Conference on Low-dimensional topology
University of Virginia, December 15-19, 2004

Ian Agol "*Rank and Heegaard genus of arithmetic 3-manifolds*

Abstract: We show that there are finitely many arithmetic 3-manifolds of Heegaard genus g, up to taking (subdihedral) cyclic covers of (semi)fibered arithmetic 3-manifolds. Assuming the short geodesic conjecture (or the Lehmer conjecture), we show that there are only finitely many closed arithmetic hyperbolic 3-manifolds with 2 generator fundamental group.

Jørgen Andersen "*Asymptotic faithfulness of the quantum $SU(n)$ representations of the mapping class groups*"

Abstract: In this talk we shall discuss our result that the sequence of projective quantum $SU(n)$ representations of the mapping class group of a closed oriented surface, obtained from the projective flat $SU(n)$-Verlinde bundles over Teichmuller space, is asymptotically faithful, that is the intersection over all levels of the kernels of these representations is trivial, whenever the genus is at least 3. For the genus 2 case, this intersection is exactly the order two subgroup, generated by the hyperelliptic involution, in the case of even degree and $n = 2$. Otherwise the intersection is also trivial in the genus 2 case. The proof makes use of the theory of Toeplitz operators and that they are asymptotically flat with respect to Hitchin's connection in the endomorphism bundle of the $SU(n)$-Verlinde bundle. - If time permits we will discuss further ramifications of our program of using asymptotics of Toeplitz operators to study the quantum $SU(n)$ theories.

Tim Cochran "*Homological Equivalence, Duality and Group Theory*"

Abstract: Suppose W is a 4-dimensional manifold with boundary M. I discuss several recent new results concerning how the fundamental groups of M and W are intricately related. One result is:

Theorem: Suppose M is zero surgery on a classical knot K and W is the exterior of a topological slice disk for K. If the degree of the Alexander polynomial of K is more than 2, then the fundamental group of W cannot be virtually solvable. (if it is 0 or 2 then it may be virtually solvable)

This type of theorem accentuates the futility of attacking the topological knot concordance problem using the currently-known technology of topological surgery.

The theorem above is true in broader situations, which will be discussed. It is true for Poincare Duality complexes.

The results discussed are based on previous work with Shelly Harvey, Kent Orr, Peter Teichner and Taehee Kim.

Jim Conant "*A chirality conjecture concerning congruence classes of Conway coefficients*"

Abstract: A knot is said to be amphicheiral if it is isotopic to its mirror image. The Conway polynomial, being invariant under mirror image, seems to have nothing to say about whether a knot is amphicheiral. However, in this talk we will show that if a knot is indeed amphicheiral, then certain mod 2 congruences seem to hold among polynomials in the coefficients. The conjectured formulas were arrived at using the theory of Vassiliev invariants, and have been verified for the first 200 amphicheiral knots. An interesting feature of the construction is a kind of formal logarithm which takes power series with integral coefficients to power series with integral coefficients, and sends multiplication to addition.

Nathan Dunfield "*Does a random 3-manifold fiber over the circle?*"

Abstract: I'll discuss the question of when a tunnel number one 3-manifold fibers over the circle — the motivation is here is to try to get some handle on the Virtual Fibration Conjecture for hyperbolic 3-manifolds. In particular, I will discuss a criterion of Brown which answers this question from a presentation of the fundamental group. I will describe how techniques of Agol, Hass, and W. Thurston can be adapted to calculate this very efficiently by using that the relator comes from an embedded curve on the boundary of a genus 2 handlebody. I will then describe some experiments which strongly suggest the answer to the question: Does a random tunnel-number one 3-manifold fiber over the circle? I will end by explaining how to prove that the the observed answer is indeed correct in one of the two cases. (joint work with Dylan Thurston, Harvard)

Eaman Eftekhary "*Filtration of Heegaard Floer homology and gluing formulas*"

Abstract: We introduce an extra filtration on the complex $\widehat{CFK}(Y,K)$ associated with a null-homologous knot K inside the three-manifold Y, denoted by $CFK_\bullet(Y,K)$, with $\bullet \in \{-,0,+\}$. This filtration will present the longitude theory $\widehat{CFL}(Y,K)$ as a subcomplex of $\widehat{CFK}(Y,K)$. The surgery exact sequences respect this filtration. Besides some basic properties of these filtered complexes, we derive a formula for CFK of the knot (Y,K) obtained by gluing the knot complements $Y_1 \setminus \mathrm{nd}(K_1)$ and $Y_2 \setminus \mathrm{nd}(K_2)$. We will also compute the filtered complex $CFK_\bullet(S^3,K)$ for an alternating knot K.

Bill Floyd "*The subdivision complex at infinity*"

Abstract: I'll discuss joint work with Cannon, Hersonsky, and Parry on Cannon's conjecture that a Gromov-hyperbolic group with space at infinity a 2-sphere is a Kleinian group. Suppose G is a torsion-free Gromov-hyperbolic group whose space at infinity is a 2-sphere. From the action of G on its space at infinity one can

construct a non-Hausdorff surface (the subdivision complex at infinity) together with a self-map (the subdivision map) on this non-Hausdorff surface. Furthermore, G is a Kleinian group if and only if there is an invariant conformal structure on the subdivision complex. This leads to the problem (which we don't understand yet) of developing a suitable Teichmueller theory for non-Hausdorff surfaces.

Stefan Friedl "*Twisted Alexander polynomials and the Thurston norm.*"

Abstract: Let M be a 3–manifold. The Thurston norm measures the size of an embedded surface representing an element in $H_2(M, \partial(M))$ Generalizing work of McMullen we show that the degrees of twisted Alexander polynomials give lower bounds for the Thurston norm. In particular we give examples of knots with trivial Alexander polynomial for which twisted Alexander polynomials give the right genus bound. Application to symplectic geometry will be discussed.

Cameron Gordon "*Knots with unknotting number 1 and Conway spheres*"

Abstract: We will describe joint work with John Luecke on knots with essential Conway spheres that have unknotting number 1. We show that if K is such a knot, then either any unknotting move for K can be isotoped to miss all the essential Conway spheres of K, or K belongs to an explicit family of knots constructed by Eudave-Munoz, or K contains a tangle summand belonging to an analogous family. This gives strong conditions on when an algebraic knot (in the sense of Conway), that is not a Montesinos knot of length 3, has unknotting number 1, and shows that there is an algorithm to decide whether or not such a knot has unknotting number 1. Another consequence is that having unknotting number 1 is invariant under mutation.

Ian Hambleton "*Signatures and Signs*"

Abstract. (joint work with Andrew Korzeniewski and Andrew Ranicki) One of the classical invariants of a symmetric bilinear form on a free abelian group is its "signature". The signature appears in topology as the signature of the "intersection form" of a manifold. In this talk I will describe a connection between the signature (mod 4) of a manifold and a new 'absolute torsion' invariant, obtained by counting + and - signs, generalizing the classical formula signature = rank + det -1 (mod 4) for unimodular symmetric bilinear forms. The new invariant is a homotopy invariant and non-trivial (e.g. for 4-manifolds), but its relation to other standard invariants is not yet clear for non-simply connected manifolds.

As an application, we can prove that the signature is multiplicative (mod 4) for any smooth fibre bundle of compatibly oriented closed manifolds.

Shelly Harvey "*Homology Cobordism of 3-Manifolds*"

Abstract: In 1964, John Stallings established an important relationship between the low-dimensional homology of a group and its lower central series. We establish a similar relationship between the low-dimensional homology of a group and its derived series. We apply this to the study of homology cobordism of 3-manifolds. Using the Cheeger-Gromov von Neumann rho invariant, for each n, we define a real valued homology cobordism invariant ρ_n. We show that, under mild hypotheses, the ρ_n take on a dense set of values and are independent for varying n. Using these invariants, we show that the successive quotients of the grope filtration of the (string) link concordance group (defined by T. Cochran, K. Orr and P. Teichner) for links with at least 2 components is infinitely generated at every stage.

Misha Kapovich "*3-dimensional Poincare duality groups*"

Mikhail Khovanov "*Matrix factorizations and link homology*"

Abstract: A matrix factorization of a polynomial f is a pair of square matrices whose product is f times the identity matrix. We'll use matrix factorizations to categorify various one-parameter specializations of the HOMFLYPT polynomial.

Taehee Kim "*Non-triviality of the Cochran-Orr-Teichner filtraion of the knot concorance group*"

Abstract : In recent years, Cochran, Orr, and Teichner established a filtration of the classical knot concordance group introduing the notion of (n)-solvability. Their theory is "abstract" in the sense that it is about 4-manifolds with only one boundary component. We construct a relative verseion of their theory. That is, we generalize their idea to the case when a 4-manifold has more than one boundary components. This induces a new equivalence relation on knots, say (n)-solvequivalence, for each half-integer n. Using this, we refine Cochran-Teichner's results on non-triviality of the Cochran-Orr-Teichner filtration and the filtration of the knot concordance group obtained by using symmetic Grope cobordism in the 4-ball.

Bruce Kleiner "*Perelman's work on Ricci flow*"

Yi-Jen Lee "*Heegaard splitting and Seiberg-Witten monopoles*"

Abstract: I'll outline a program to relate the Heegaard Floer homologies of Ozsvath-Szabo, and Seiberg-Witten-Floer homologies as defined by Kronheimer-Mrowka. The center-piece of this program is the construction of an intermediate version of Floer theory, which exhibits characteristics of both theories.

Tao Li "*Heegaard surfaces and measured laminations*"

Abstract: We will discuss different Heegaard splittings of a 3-manifold and the connection between Heegaard surfaces and measured laminations. Using measured laminations, we give a proof of the so-called generalized Waldhausen conjecture, which says that an orientable irreducible atoroidal 3-manifold has only finitely many Heegaard splittings in each genus, up to isotopy. We will also show that a closed non-Haken 3-manifold has only finitely many irreducible Heegaard splittings, up to isotopy.

Olga Plamenevskaya "*Transverse links, Khovanov homology and contact structures on branched double covers*"
Abstract: Given a transverse link in the standard contact 3-sphere, we introduce an invariant which is an element of the Khovanov homology of the link and use it to get a bound on the self-linking number of the transverse link.

Recently, Ozsvath and Szabo established the relation between Khovanov homology of the link and the Heegaard Floer homology of the branched double cover. Using this relation, we conjecture that our invariant corresponds to the Heegaard Floer contact invariant of the double cover branched over the transverse link. To support this conjecture, we compute the contact invariant in certain cases.

Frank Quinn:

"*Characteristic structures and behaviors in high and low dimensions.*"
Abstract: By 60 years ago definitions had solidified for four main types of manifolds: smooth, PL, topological, and homology. Methods were quite different for each and it seemed possible that there would be four distinct branches of manifold topology. The basic theories are still different, but by 20 years ago deep structural commonalities in high dimensions (surgery, bundle theories, handlebodies, s-cobordisms etc) had led to a view of "manifoldness" as being characterized by these commonalities. Different structures (smooth, PL etc.) appear almost as decorations controlled by bundle theory.

Manifolds not fitting the high-dimensional pattern are those of dimension 2 and 3 and smooth 4-manifolds. These are "low" by default since there is not yet a characteristic "low dimensional" behavior. There are strong commonalities between 2 and 3 (geometrization and the structure of codimension 1 objects) but these will not extend in any straightforward way to 4, and they seem to fall short of providing a full picture in dimension 3. Smooth 4-manifolds remain mysterious. Is there any deep unity to the low dimensions or must they be divided into cases?

"*Are topological 4-manifolds high-dimensional?*"
Abstract: The development of 4-dimensional topology due largely to Freedman showed that many 4-manifolds share the structural patterns of higher dimensional manifolds. These methods failed to work for manifolds with "large" fundamental groups, and Freedman conjectured that there is some residual "low dimensional" behavior in these manifolds. However there is now reason to suspect that they are in fact high dimensional. I will outline a still-provisional proof of the topological surgery conjecture. It is quite indirect, going through controlled surgery on Poincare spaces, homology manifolds, and the resolution theorem, so there may be low-dimensional obstructions to success of the earlier approaches.

"*Dual decompositions and s-cobordisms of 4-manifolds*"
Abstract: Dual decompositions are structures on smooth 4-manifolds that exploit

a weak disk embedding theorem. The ideal disk embedding theorem provides an embedded disk with specified boundary curve on the boundary of the manifold. The weak version permits the boundary curve to change up to homotopy. One application is a decomposition of a smooth s-cobordism of 4-manifolds as a union of a product and a "core", an s-cobordism of a neighborhood of a graph. We sketch a possible route to the topological s-cobordism theorem by connecting with the (known) theorem for s-cobordisms controlled over a graph.

Jake Rasmussen "*Khovanov homology and embedded surfaces*"
Abstract: Khovanov homology is a functor from the category whose objects are knots in S^3 and whose morphisms are cobordisms (smooth embedded surfaces in $S^3 \times [0,1]$) to the category of abelian groups. Although this functor seems rather difficult to understand, there is a slight perturbation of it due to Lee which turns out to be exactly solvable. I'll describe how this fact can be applied to compute Khovanov's invariants for closed surfaces and to study the smooth four-ball genus of knots in S^3.

Lev Rozansky "*Matrix factorizations and the categorification of the SU(N) HOMFLY polynomial*"
Abstract: This is a joint work with M. Khovanov. The Jones and HOMFLY polynomials of links are mysterious invariants: their most natural topological definition comes from the Chern-Simons-Witten path integral, but their polynomial nature becomes transparent only within the combinatorial definition, which lacks a clear topological interpretation. M. Khovanov suggested that these polynomials are graded Euler characteristics of chain complexes of Z-graded vector spaces, which are link invariants up to a homotopy. To a cobordism of links one should associate a morphism of these chain complexes.

The complexes are constructed along the lines of the Murakami-Ohtsuki-Yamada formula for the HOMFLY polynomial. A picture of an n-component link is converted into a set of 2^n 3-valent graphs G, and to each graph we associate a Z-graded vector space H(G). The chain complex spaces are the direct sums of the spaces H(G), while the differentials come from the presentation of the spaces H(G) as morphisms in the category of matrix factorizations.

Danny Ruberman "*Rohlin's invariant and gauge theory*"

Rob Schneiderman "*Whitney towers, gropes and links*"

Zoltan Szabó:

"*Exotic smooth structures on* $CP^2\#n\overline{CP^2}$"

"*Holomorphic disks and Heegaard diagrams*"

"*Some applications of Heegaard Floer homology*"