

Some
mathematical
work of Bob
Stong

Outline

Historical
setting

Some early
publications

The connective
covers paper

The
Stong-Hattori
theorem

Cobordism of
maps

Notes on
Cobordism
Theory

Some later
papers

Ochanine's
theorem

Landweber-
Stong
1988

LRS 1993

The end

Some early and middle mathematical work of Bob Stong

Doug Ravenel

University of Rochester

November 10, 2007

Bob's early career

- 1954-59: B.S. and M.A, University of Oklahoma
- 1959-62: M.S. and Ph.D. at University of Chicago under Dick Lashof
- 1962-64: Lieutenant in U.S. Army
- 1964-66: NSF Postdoctoral Fellow at Oxford
- 1966-68: Instructor at Princeton
- 1968 to present: University of Virginia

Some
mathematical
work of Bob
Stong

Outline

Historical
setting

Some early
publications

The connective
covers paper

The
Stong-Hattori
theorem

Cobordism of
maps

Notes on
Cobordism
Theory

Some later
papers

Ochanine's
theorem

Landweber-
Stong
1988

LRS 1993

The end

The mathematical landscape in 1962

- Thom's work on cobordism, including the computation of $\pi_*(MO)$, the unoriented cobordism ring, 1954
- Lectures (at Princeton) on characteristic classes by Milnor, 1957 (later a book by Milnor and Stasheff, 1974)
- Bott periodicity, 1959
- The Adams spectral sequence, 1959, and Hopf invariant one, 1961
- The Riemann-Roch theorem of Atiyah-Hirzebruch, 1959
- Computations of $\pi_*(MU)$ (the complex cobordism ring) by Milnor and Novikov and of $\pi_*(MSO)$ (the oriented cobordism ring) by Wall, 1960
- Lectures (at Harvard) on K-theory by Bott, 1962, and Atiyah, 1964

Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

What lay in the future

- Brown-Peterson theory (*BP*), 1967
- The Adams-Novikov spectral sequence, 1967
- Quillen's theorem on the connection between formal groups laws and complex cobordism, 1969
- Morava K-theory, roughly 1973
- Landweber exact functor theorem, 1976
- Ochanine's elliptic genus, 1987

Some
mathematical
work of Bob
Stong

Outline

Historical
setting

Some early
publications

The connective
covers paper

The
Stong-Hattori
theorem

Cobordism of
maps

Notes on
Cobordism
Theory

Some later
papers

Ochanine's
theorem

Landweber-
Stong
1988

LRS 1993

The end

Mod 2 cohomology of the connective covers of BO and BU

“Determination of $H^*(BO(k, \dots, \infty), \mathbb{Z}_2)$ and $H^*(BU(k, \dots, \infty), \mathbb{Z}_2)$ ” appeared in the AMS Transactions in 1963.

It was a tour de force computation with the Serre spectral sequence and the Steenrod algebra.

It was the first determination of the cohomology of an infinite delooping of an infinite loop space, other than the Eilenberg-Mac Lane spectrum.

It appeared while Bob was in the Army.

Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

Mod 2 cohomology of the connective covers of BO and BU , 1963

The first three connective covers of BO are

$$BO(2, \infty) = BSO$$

$$BO(3, \infty) = BO(4, \infty) = BSpin$$

$$BO(5, \infty) = \cdots = BO(8, \infty) = BString$$

The map from $H^*(BO)$ to the cohomology of each of these is onto.

This is not true of the higher connective covers.

Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

Mod 2 cohomology of the connective covers of BO and BU , 1963

The first two connective covers of BU are

$$\begin{aligned}BU(3, \infty) &= BU(4, \infty) = BSU \\BU(5, \infty) &= BU(6, \infty) = BU(6)\end{aligned}$$

The integral cohomology of each of them is torsion free, which is not true of the higher connective covers.

The Thom spectra $MString$ and $MU(6)$ both figure in the theory of topological modular forms.

Some
mathematical
work of Bob
Stong

Outline

Historical
setting

Some early
publications

The connective
covers paper

The
Stong-Hattori
theorem

Cobordism of
maps

Notes on
Cobordism
Theory

Some later
papers

Ochanine's
theorem

Landweber-
Stong
1988

LRS 1993

The end

Mod 2 cohomology of the connective covers of BO and BU , 1963, continued

The main result of the paper implies that

$$H^*(bo; \mathbf{Z}/2) = A \otimes_{A(1)} \mathbf{Z}/2$$

where $A(1)$ denotes the subalgebra of the mod 2 Steenrod algebra A generated by Sq^1 and Sq^2 ,

and

$$H^*(bu; \mathbf{Z}/2) = A \otimes_{Q(1)} \mathbf{Z}/2$$

where $Q(1)$ denotes the subalgebra generated by Sq^1 and $[Sq^1, Sq^2]$.

These results are indispensable for future work on Spin cobordism and connective real and complex K-theory.

Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

Mod 2 cohomology of the connective covers of BO and BU , 1963, methods used

It was known that

$$H^*(BO; \mathbf{Z}/2) = \mathbf{Z}/2[w_1, w_2, \dots]$$

where $w_i \in H^i$ is the i th Stiefel-Whitney class.

Bob replaced the generator w_i with $\theta_i \in H^i$, which he defined as the image of a certain Steenrod operation acting on w_{2^m} , where $m = \alpha(j - 1)$, with $\alpha(j)$ being the number of ones in the dyadic expansion of j .

Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

Mod 2 cohomology of the connective covers of BO and BU , 1963, methods used

For example we have $\theta_1 = w_1$,

$$\theta_2 = w_2 \xrightarrow{Sq^1} \theta_3 \xrightarrow{Sq^2} \theta_5 \xrightarrow{Sq^4} \theta_9 \xrightarrow{Sq^8} \dots$$

and

$$\theta_4 = w_4 \xrightarrow{Sq^3} \theta_7 \xrightarrow{Sq^6} \theta_{13} \xrightarrow{Sq^{12}} \theta_{25} \xrightarrow{Sq^{24}} \dots$$

$$\begin{array}{c} \swarrow Sq^2 \\ \theta_6 \xrightarrow{Sq^5} \theta_{11} \xrightarrow{Sq^{10}} \theta_{21} \xrightarrow{Sq^{20}} \dots \end{array}$$

$$\begin{array}{c} \swarrow Sq^4 \\ \theta_{10} \xrightarrow{Sq^9} \theta_{19} \xrightarrow{Sq^{18}} \dots \end{array}$$

$$\begin{array}{c} \swarrow Sq^8 \\ \theta_{18} \xrightarrow{Sq^{17}} \dots \end{array}$$

$$\begin{array}{c} \swarrow Sq^{16} \\ \dots \end{array}$$

Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

Mod 2 cohomology of the connective covers of BO and BU , 1963, methods used

He also makes use of some exact sequences of A -modules discovered by Toda. They can be obtained by applying the functor $A \otimes_{A(1)} (\cdot)$ to the following sequences exact of $A(1)$ -modules.

For the BU computation,

$$\begin{array}{c} A/A(Sq^3, Sq^1) \\ \uparrow \\ A/ASq^1 \\ \uparrow \\ \Sigma^3 A/A(Sq^3, Sq^1) \end{array}$$

Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

Mod 2 cohomology of the connective covers of BO and BU , 1963, methods used

$$\begin{array}{c}
 A/A(Sq^3, Sq^1) \\
 \uparrow \\
 A/ASq^1 \\
 \uparrow \\
 \Sigma^3 A/A(Sq^3, Sq^1)
 \end{array}$$

This short exact sequence defines (via a change-of-rings isomorphism relating $A(1)$ and $Q(1)$) an element in

$$\text{Ext}_{Q(1)}^{1,3}(\mathbf{Z}/2, \mathbf{Z}/2)$$

corresponding to the complex form of Bott periodicity.

Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

Mod 2 cohomology of the connective covers of BO and BU , 1963, methods used

The following 6-term exact sequence defines an element in

$$\mathrm{Ext}_{A(1)}^{4,12}(\mathbf{Z}/2, \mathbf{Z}/2)$$

corresponding to the real form of Bott periodicity.

Some
mathematical
work of Bob
Stong

Outline

Historical
setting

Some early
publications

The connective
covers paper

The
Stong-Hattori
theorem

Cobordism of
maps

Notes on
Cobordism
Theory

Some later
papers

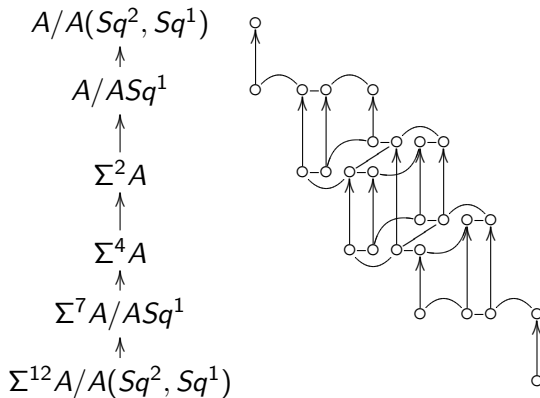
Ochanine's
theorem

Landweber-
Stong
1988

LRS 1993

The end

Mod 2 cohomology of the connective covers of BO and BU , 1963, methods used



Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

Mod 2 cohomology of the connective covers of BO and BU , 1963, methods used

Within this diagram are four modules of interest:

$$\begin{array}{ccc}
 A/A(Sq^1, Sq^2) \circ & & A/ASq^2 \quad \circ\text{---}\circ\text{---}\circ \\
 A/ASq^3 \quad \circ\text{---}\circ\text{---}\circ\text{---}\circ & & A/A(Sq^1, Sq^5) \quad \circ\text{---}\circ\text{---}\circ
 \end{array}$$

which we denote by M_0 , M_1 , M_2 and M_4 .

$M_s = H^*(bo\langle s \rangle)$, the mod 2 cohomology of the stable connective cover in the diagram

$$\begin{array}{ccccccc}
 bo = bo\langle 0 \rangle & \longleftarrow & bo\langle 1 \rangle & \longleftarrow & bo\langle 2 \rangle & \longleftarrow & bo\langle 4 \rangle \\
 \downarrow & & \downarrow & & \downarrow & & \\
 K(\mathbf{Z}, 0) & & K(\mathbf{Z}/2, 1) & & K(\mathbf{Z}/2, 2) & &
 \end{array}$$

Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

Mod 2 cohomology of the connective covers of BO and BU , statement of main result

Theorem

Let $s = 0, 1, 2$ or 4 , $8k + s > 0$,
and $K_{8k+s} = K(\pi_{8k+s}(BO), 8k + s)$.

Then

$$H^*(BO(k, \dots, \infty)) = P(\theta_i: \alpha(i-1) \geq 4k + s' - 1) \\ \otimes H^*(K_{8k+s}) / (AQ_{s'} l_{8k+s})$$

where

$$(s', Q_s) = \begin{cases} (0, Sq^2) & \text{for } s = 0 \\ (1, Sq^2) & \text{for } s = 1 \\ (2, Sq^3) & \text{for } s = 2 \\ (3, Sq^5) & \text{for } s = 4 \end{cases}$$

The image of $H^*(K_{8k+s})$ is an unstable version of M_s .

Some
mathematical
work of Bob
Stong

Outline

Historical
setting

Some early
publications

The connective
covers paper

The
Stong-Hattori
theorem

Cobordism of
maps

Notes on
Cobordism
Theory

Some later
papers

Ochanine's
theorem

Landweber-
Stong
1988

LRS 1993

The end

The Stong-Hattori theorem

This was the subject of Bob's two papers "Relations among characteristic numbers I and II," which appeared in *Topology* in 1965 and 1966, while he was at Oxford.

He proved it for cobordism theories associated with five classical groups: U , SU , SO , $Spin$ and $Spin^c$.

Hattori published a paper in 1966 reproving the theorem in the complex case.

Some
mathematical
work of Bob
Stong

Outline

Historical
setting

Some early
publications

The connective
covers paper

The
Stong-Hattori
theorem

Cobordism of
maps

Notes on
Cobordism
Theory

Some later
papers

Ochanine's
theorem

Landweber-
Stong
1988

LRS 1993

The end

What the theorem says in the complex case

A complex manifold M of dimension n (meaning real dimension $2n$) comes equipped with a map $f : M \rightarrow BU$ classifying its stable normal bundle.

This gives us normal Chern classes $f^*(c_i) \in H^{2i}(M)$ for $1 \leq i \leq n$.

Any degree n monomial c^J in these can be evaluated on the fundamental homology class of M , and we get a **Chern number**

$$\langle c^J, [M] \rangle \in \mathbf{Z}.$$

Chern numbers were shown by Milnor to be complete cobordism invariants, i.e., M is a boundary iff all of its Chern numbers vanish.

Some
mathematical
work of Bob
Stong

Outline

Historical
setting

Some early
publications

The connective
covers paper

The
Stong-Hattori
theorem

Cobordism of
maps

Notes on
Cobordism
Theory

Some later
papers

Ochanine's
theorem

Landweber-
Stong
1988

LRS 1993

The end

What the theorem says in the complex case, continued

Equivalently, the classifying map f of the normal bundle on M gives us an element $f_*([M]) \in H_{2n}(BU)$ which depends only on the cobordism class of M .

This leads to a monomorphic ring homomorphism

$$\text{complex cobordism ring} = \pi_*(MU) \rightarrow H_*(BU).$$

Composing this with the Thom isomorphism from $H_*(BU)$ to $H_*(MU)$ gives us the stable Hurewicz map

$$\eta : \pi_*(MU) \rightarrow H_*(MU)$$

Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

What the theorem says in the complex case, 3

The question that Stong addressed is

What is the image of $\eta : \pi_*(MU) \rightarrow H_*(MU)$?

Two facts were already known about this:

- (i) Tensoring both side with the rationals converts η into an isomorphism, so the image of η has locally finite index.
- (ii) The Atiyah-Hirzebruch Riemann-Roch theorem (which was proved using K -theory) says that certain integrality relations must hold among these numbers.

Some
mathematical
work of Bob
Stong

Outline

Historical
setting

Some early
publications
The connective
covers paper

The
Stong-Hattori
theorem

Cobordism of
maps

Notes on
Cobordism
Theory

Some later
papers

Ochanine's
theorem

Landweber-
Stong
1988

LRS 1993

The end

What the theorem says in the complex case, 4

Equivalently, the image of

$$\eta : \pi_*(MU) \rightarrow H_*(MU)$$

is contained in a certain subgroup $B^U \subset H_*(MU)$ defined by Atiyah-Hirzebruch.

Bob showed that the image is precisely this subgroup.

He did it by constructing certain complex manifolds with appropriate Chern numbers.

Some
mathematical
work of Bob
Stong

Outline

Historical
setting

Some early
publications

The connective
covers paper

The
Stong-Hattori
theorem

Cobordism of
maps

Notes on
Cobordism
Theory

Some later
papers

Ochanine's
theorem

Landweber-
Stong
1988

LRS 1993

The end

A reformulation of the Stong-Hattori theorem

The image of the K -theoretic Hurewicz homomorphism

$$\pi_*(MU) \rightarrow K_*(MU)$$

is a direct summand. This is not true in ordinary homology.

In the other four cases, MSU , MSO , $MSpin$ and $MSpin^c$, Bob proved similar statements using KO_* instead of K_* .

Some
mathematical
work of Bob
Stong

Outline

Historical
setting

Some early
publications

The connective
covers paper

The
Stong-Hattori
theorem

Cobordism of
maps

Notes on
Cobordism
Theory

Some later
papers

Ochanine's
theorem

Landweber-
Stong
1988

LRS 1993

The end

Cobordism of maps, *Topology 1966*

Two maps $f_1 : M_1 \rightarrow N_1$ and $f_2 : M_2 \rightarrow N_2$ (where M_1 and M_2 are closed m -dimensional manifolds, while N_1 and N_2 are closed n -dimensional manifolds) are *cobordant* if there is a diagram

$$\begin{array}{ccccc} M_1 & \hookrightarrow & V & \longleftarrow & M_2 \\ \downarrow f_1 & & \downarrow g & & \downarrow f_2 \\ N_1 & \hookrightarrow & W & \longleftarrow & N_2 \end{array}$$

where V is an $(m + 1)$ -manifold whose boundary is $M_1 \amalg M_2$, and W is an $(n + 1)$ -manifold whose boundary is $N_1 \amalg N_2$.

Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

Cobordism of maps, *Topology 1966*, continued

Assume that all manifolds in sight have compatible G -structures in their stable normal bundles for $G = O, SO, U$, etc.

Let $\Omega^G(m, n)$ denote the group of cobordism classes of maps as defined above, and let MG denote the Thom spectrum associated with G .

Bob showed that

$$\begin{aligned}\Omega^G(m, n) &\cong \lim_{r \rightarrow \infty} MG_n(\Omega^{r+m} MG_{r+n}) \\ &\cong MG_n(\Omega^\infty MG_{n-m})\end{aligned}$$

The cobordism groups of maps are the bordism groups of the spaces in the Ω -spectrum for MG .

He described these groups explicitly for $G = O$.

Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

Bob's book, NOTES ON COBORDISM THEORY, Princeton 1968

- One stop shopping for all your cobordism needs
- Detailed table of contents available online at conference website.
- A boon to graduate students
- Retail price \$7.50

Some
mathematical
work of Bob
Stong

Outline

Historical
setting

Some early
publications

The connective
covers paper

The
Stong-Hattori
theorem

Cobordism of
maps

Notes on
Cobordism
Theory

Some later
papers

Ochanine's
theorem

Landweber-
Stong
1988

LRS 1993

The end

Fast forward 20 years:

Ochanine's theorem on elliptic genera, 1987

Theorem

Suppose we have a homomorphism $\varphi : \Omega_*^{\text{SO}} \rightarrow \Lambda$, from the oriented bordism ring to a commutative unital \mathbb{Q} -algebra. Then it vanishes on all manifolds of the form $\mathbf{CP}(\xi)$ with ξ an even-dimensional complex vector bundle over a closed oriented manifold if and only if the logarithm

$$g(u) = \sum_{n \geq 0} \frac{\varphi(\mathbf{CP}^{2n})}{2n+1} u^{2n+1}$$

of the formal group law of φ is given by an elliptic integral of the first kind, i.e., by

$$g(u) = \int_0^u \frac{dz}{\sqrt{P(z)}}, P(z) = 1 - 2\delta z^2 + \varepsilon z^4, \delta, \varepsilon \in \Lambda.$$

Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

Ochanine's theorem on elliptic genera, continued

Such a homomorphism $\varphi \Omega_*^{\text{SO}} \rightarrow \Lambda$ is now called an **elliptic genus**.

If $\delta = \varepsilon = 1$, then φ is the signature.

If $\varepsilon = 0$ and $\delta = -\frac{1}{8}$, then φ is the \hat{A} -genus.

We now know that Λ need only be an algebra over the ring $\mathbf{Z}[1/2][\delta, \varepsilon]$, which can be interpreted as a ring of modular forms.

The manifolds on which φ vanishes admit semi-free S^1 -actions (since $\mathbf{C}P^{2n-1}$ does), and $\varphi(M)$ is an obstruction to the existence of such an action.

In the past 20 years there has been a lot of interest in interpreting such a genus geometrically or analytically.

Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

The 1988 paper of Landweber and Stong

Circle actions on Spin manifolds and characteristic numbers appeared in TOPOLOGY in 1988.

Bob and Peter gave a formula for an elliptic genus with values in $\mathbf{Q}[[q]]$ in terms of *KO*-characteristic classes,

$$\varphi(M) = \sum_{k=0}^{\infty} \langle \hat{A}(M) \text{ch}(\rho_k(TM)), [M] \rangle q^k$$

where $\hat{A}(M)$ the total \hat{A} -class of the tangent bundle of M , $\text{ch}(E)$ is the Chern character of the complexification of E and $\rho_k(TM)$ is a certain element in $KO^*(M) \otimes \mathbf{Q}$, now known to be integral.

Some
mathematical
work of Bob
Stong

Outline

Historical
setting

Some early
publications

The connective
covers paper

The
Stong-Hattori
theorem

Cobordism of
maps

Notes on
Cobordism
Theory

Some later
papers

Ochanine's
theorem

Landweber-
Stong
1988

LRS 1993

The end

The 1993 paper of Landweber, Ravenel and Stong

Periodic cohomology theories defined by elliptic curves appeared in the proceedings of the Čech centennial conference of 1993.

In it we considered the genus defined above by Ochanine with values in $\mathbf{Z}[1/2][[\delta, \varepsilon]]$, regarded as a homomorphism out of the complex cobordism ring MU_* .

Whenever one has an R -valued genus φ on MU_* , one can ask if the functor

$$X \mapsto MU^*(X) \otimes_{\varphi} R$$

is a cohomology theory, i.e., whether it has appropriate exactness properties.

The **Landweber Exact Functor Theorem** of 1976 gives explicit criteria for such exactness.

Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

LRS, continued

The Ochanine genus $\varphi : MU_* \rightarrow \mathbf{Z}[1/2][\delta, \varepsilon]$ does **not** satisfy these criteria.

We showed that it becomes Landweber exact after inverting either ε or $\delta^2 - \varepsilon$.

This means that if R is the ring obtained from $\mathbf{Z}[1/2][\delta, \varepsilon]$ by inverting one or both of these elements, then the functor

$$X \mapsto MU^*(X) \otimes_{\varphi} R$$

is a cohomology theory and therefore representable by the **elliptic cohomology spectrum** Ell .

Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

A homomorphism $\varphi : MU_* \rightarrow R$ is also equivalent (by Quillen's theorem) to a 1-dimensional formal group law over R .

When φ is the Ochanine genus, we get the formal group law associated with the elliptic curve defined by the Jacobi quartic,

$$y^2 = 1 - 2\delta x^2 + \varepsilon x^4.$$

Some
mathematical
work of Bob
Stong

Outline

Historical
setting

Some early
publications

The connective
covers paper

The
Stong-Hattori
theorem

Cobordism of
maps

Notes on
Cobordism
Theory

Some later
papers

Ochanine's
theorem

Landweber-
Stong
1988

LRS 1993

The end

LRS and other elliptic curves

The same method entitles us to construct multiplicative homology theories with coefficient rings

$$\mathbf{Z}[\frac{1}{6}][g_2, g_3, \Delta^{-1}] \quad \text{where } \Delta = g_2^3 - 27g_3^2$$

corresponding to the elliptic curve defined by the Weierstrass equation

$$y^2 = 4x^3 - g_2x - g_3,$$

and the ring

$$\mathbf{Z}[a_1, a_2, a_3, a_4, a_6, \Delta^{-1}]$$

where Δ is the discriminant of the equation

$$y^2 + a_1xy + a_3y = x^3 + a_2x^2 + a_4x + a_6.$$

Some mathematical work of Bob Stong

Outline

Historical setting

Some early publications

The connective covers paper

The Stong-Hattori theorem

Cobordism of maps

Notes on Cobordism Theory

Some later papers

Ochanine's theorem

Landweber-Stong 1988

LRS 1993

The end

LRS and modular forms

$\mathbf{C}[\delta, \varepsilon]$ is naturally isomorphic to the ring $M_*(\Gamma_0(2))$ of modular forms for the group $\Gamma_0(2) \subset \mathrm{SL}_2(\mathbf{Z})$, with δ and ε having weights 2 and 4, respectively.

This isomorphism sends the subring $M_* = \mathbf{Z}[\frac{1}{2}][\delta, \varepsilon]$ to the modular forms whose q -expansions at the cusp $\tau = \infty$ have coefficients in $\mathbf{Z}[\frac{1}{2}]$.

Moreover, the localizations $M_*[\Delta^{-1}]$, $M_*[\varepsilon^{-1}]$ and $M_*[(\delta^2 - \varepsilon)^{-1}]$ correspond to the rings of modular functions which are holomorphic on \mathcal{H} (the complex upper half plane), $\mathcal{H} \cup \{0\}$ and $\mathcal{H} \cup \{\infty\}$, respectively, and whose q -expansions have coefficients in $\mathbf{Z}[\frac{1}{2}]$.

Some
mathematical
work of Bob
Stong

Outline

Historical
setting

Some early
publications

The connective
covers paper

The
Stong-Hattori
theorem

Cobordism of
maps

Notes on
Cobordism
Theory

Some later
papers

Ochanine's
theorem

Landweber-
Stong
1988

LRS 1993

The end

The end

Enjoy your retirement, Bob!

Some
mathematical
work of Bob
Stong

Outline

Historical
setting

Some early
publications

The connective
covers paper

The
Stong-Hattori
theorem

Cobordism of
maps

Notes on
Cobordism
Theory

Some later
papers

Ochanine's
theorem

Landweber-
Stong
1988

LRS 1993

The end